Short Course Syllabus – University of Bologna

September 2025

Title: Fundamental Properties of Harmonic Functions

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This 10-hour course, held at the Department of Mathematics of the University of Bologna, covers the fundamental properties of Harmonic functions. The primary objective is to establish the foundational concepts and tools that offer insight into current research questions in the field of Elliptic Partial Differential Equations. The course will meet on September 8th, 15th, 18th, 22nd, and 29th, 2025.

Upon completion of the classes, students will be provided with a set of exercises pertaining to the discussed topics. Final grades of A, B, C, and D will be awarded based on student performance.

The course will cover the following key topics:

1. Mean value formulas;
2. Interior gradient estimates
3. Basic Liouville Theorems
4. Weak Maximum Principle
5. Strong Maximum Principe
6. Boundary gradient estimates for Poisson equation
7. Weak Harnack Inequality
8. Harnack Inequality
9. Local Maximum Principle
10. Barriers based on Fundamental Solutions

11. Quantitative version of the Hopf-Oleinik Lemma and Unique Continuation
12. Poisson Integral Formula in Balls - Basic Existence Result
13. Removability of singularities for Harmonic functions
14. Viscosity solutions, Weyl's Lemma and equivalence of solutions
15. Classifications of nonnegative in half spaces
16. Boundary Harnack Inequality and its consequences